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A New Interpretation of Vaidya Radiation

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Abstract

If Vaidya radiation is taken to be a combination of an electromagnetic energy flux together with that of a null fluid it is shown that the singularity in the current density along an axis, which arises in the absence of a null field, is removed.

1. *Introduction*

Vaidya's solution for the radiating mass particle involves a singularity in the electromagnetic current density when this solution is interpreted in terms of an electromagnetic field. The object of this note is to show that this singularity may be removed if Vaidya radiation is considered as composed of two parts, an energy flow of electromagnetic radiation plus an energy flux due to that of a null fluid. We are indebted to Professor W. B. Bonnor for this idea. Firstly, some preliminary information will be given.

2. *The Metric and Electromagnetic Field Equations*

It has been shown (Goodinson & Newing, 1970) that the metric for any space-time admitting a null current can be expressed in the form,

$$ds^2 = \alpha\, dx^{12} + 2\, dx^0\, dx^1 + 2\, dx^1(\beta\, dx^2 + \gamma\, dx^3) - k(dx^{2^2} + dx^{3^2})$$

where α, β, γ are independent of x^0.

Now if L_α is the propagation vector of a null electromagnetic field, then in the absence of matter and in suitable units the Ricci tensor of the space-time may be expressed in the form

$$R_{\alpha\beta} = -L_\alpha L_\beta = -E_{\alpha\beta}$$

With reference to Goodinson & Newing (1970), a tetrad of null vectors can be constructed in the space-time where the vectors L_α, M_α can be taken to be

$$L_\alpha = \lambda \delta_\alpha{}^1, \qquad L^\alpha = \lambda \delta_0{}^\alpha$$

$$M_\alpha = \frac{1}{\sqrt{(2k)}}\{(\beta - i\gamma)\,\delta_\alpha{}^1 - k(\delta_\alpha{}^2 - i\delta_\alpha{}^3)\}$$

$$M^\alpha = \frac{1}{\sqrt{(2k)}}(\delta_2{}^\alpha - i\delta_3{}^\alpha)$$

The current density vector J^α is expressed in the form

$$J^\alpha = \frac{\lambda \exp(i\phi)}{\sqrt{(2k)}}\{(\chi + i\phi)_{,2} - i(\chi + i\phi)_{,3}\}\,\delta_0{}^\alpha$$

where $\chi = \log(\lambda\sqrt{k})$ and ϕ is the complexion parameter of the electromagnetic field.

The condition that J^α be real, may be satisfied by taking

$$\cos\phi = \exp(-\chi)\,F_{,2}, \qquad \sin\phi = \exp(-\chi)\,F_{,3}$$

where F is some function of x^1, x^2, x^3 subject to the condition

$$(F_{,2})^2 + (F_{,3})^2 = \lambda^2 k \tag{2.1}$$

J^α then becomes,

$$J^\alpha = \frac{1}{\sqrt{(2)}\,.\,k}(F_{,22} + F_{,33})\,\delta_0{}^\alpha \tag{2.2}$$

The parameters α, β, γ, k defining the space-time and the electromagnetic parameter λ must be such that the gravitational field equations

$$R_{\alpha\beta} = -\lambda^2\,\delta_\alpha{}^1\,\delta_\beta{}^1$$

are satisfied.

As in Section 4 of Goodinson & Newing (1970) the case when

$$\alpha = 1 - \frac{2m}{x^0}, \qquad \beta = 0 = \gamma, \qquad k = x^{02}\,\mathrm{sech}^2\,x^2, \qquad (m = m(x^1))$$

produced the Vaidya radiation solution.

The main results concerning this solution are:

$$R_{\alpha\beta} = \frac{2m_1}{x^{02}}\,\delta_\alpha{}^1\,\delta_\beta{}^1, \qquad \text{where } m_1 = dm/dx^1$$

$$(F_{,2})^2 + (F_{,3})^2 = -2m_1\,\mathrm{sech}^2\,x^2$$

$$J^\mu = \frac{\cosh^2 x^2}{\sqrt{(2)}\,.\,(x^0)^2}(F_{,22} + F_{,33})\,\delta_0{}^\mu \tag{2.3}$$

$$k = x^{02}\,\mathrm{sech}^2\,x^2$$

Now by using the coordinate transformations,

$$x^0 = r, \qquad x^1 = u, \qquad \tanh x^2 = \cos\theta, \qquad x^3 = \phi$$

the above expression for J^μ becomes

$$J^\mu = \left(\frac{\sqrt{(-m_1)}}{r^2}\nabla^2 \Lambda\right)\delta_0{}^\mu \tag{2.4}$$

where ∇^2 is the usual Laplacian in spherical polar coordinates, and Λ is a function of θ and ϕ such that $\Lambda_{,\theta}{}^2 + \Lambda_{,\phi}{}^2 \operatorname{cosec}^2\theta = 1$.

The Vaidya solution, namely $\Lambda = \cos^{-1}\{\sin\theta\sin\phi\}$ makes J^μ as given by equation (2.4) infinite along the axis, $\theta = 0$. The following section shows how this singularity in J^μ can be removed.

3. *Decomposition of the Ricci Tensor*

Consider the expression

$$R_{\mu\nu} = R_{\mu\nu}\cos^2\theta + R_{\mu\nu}\sin^2\theta$$

or, alternatively, using the transformation mentioned in the preceding section, i.e., $\cos\theta = \tanh x^2$, $R_{\mu\nu}$ can be written

$$R_{\mu\nu} = R_{\mu\nu}(\tanh^2 x^2 + \operatorname{sech}^2 x^2) \tag{3.1}$$

Let us assume we have a Vaidya radiation flow along the x^1-direction, then $R_{\mu\nu}$ can be taken to be of the form $R_{\mu\nu} = -p^2\delta_\mu{}^1\delta_\nu{}^1$ where $p^2 = -2m_1/x^{02}$ as in Section 2.

Therefore (3.1) becomes

$$R_{\mu\nu} = -p^2(\operatorname{sech}^2 x^2 + \tanh^2 x^2)\delta_\mu{}^1\delta_\nu{}^1 \tag{3.2}$$

Interpreting the first term as an electromagnetic null field contribution, and the latter as that from a null fluid (Bonnor, 1970) then in the notation of Section 2, equation (3.2) can be written as,

$$R_{\mu\nu} = -\lambda^2\delta_\mu{}^1\delta_\nu{}^1 - \sigma^2\delta_\mu{}^1\delta_\nu{}^1$$

where $\lambda = p\operatorname{sech} x^2$ and $\sigma = p\tanh x^2$, σ representing the null fluid contribution.

Equation (2.1) with $k = x^{02}\operatorname{sech}^2 x^2$ and $\lambda^2 = p^2\operatorname{sech}^2 x^2$ becomes

$$\begin{aligned}(F_{,2})^2 + (F_{,3})^2 &= p^2 x^{02}\operatorname{sech}^4 x^2\\ &= -2m_1\operatorname{sech}^4 x^2\end{aligned}$$

Taking F independent of the x^3 coordinate, i.e., $F_{,3} = 0$, then $F_{,2}$ can be expressed as

$$F_{,2} = \sqrt{(-2m_1)}\operatorname{sech}^2 x^2$$

and hence

$$F_{,22} = -2\sqrt{(-2m_1)}\operatorname{sech}^2 x^2 . \tanh x^2$$

Inserting this latter expression into (2.3) gives

$$J^\mu = -\frac{2\sqrt{(-m_1)}}{x^{02}} \tanh x^2 . \delta_0{}^\mu$$

and since $x^0 = r$, $\tanh x^2 = \cos\theta$, then

$$J^\mu = \frac{-2\sqrt{(-m_1)}}{r^2} . \cos\theta . \delta_0{}^\mu \tag{3.3}$$

This expression for J^μ is clearly finite for $\theta = 0$ and so by interpreting the energy flux as a combination of that due to electromagnetic radiation, together with that of a null fluid, the θ-singularity in J^μ, present in equation (2.4) is removed.

Also, since $F_{,3} = 0$ the complexion parameter ϕ may be taken to be zero and the electromagnetic field tensor $F^{\alpha\beta}$, where $F^{\alpha\beta}$ is the real part of $(L^\alpha M^\beta - L^\beta M^\alpha)$ is independent of θ in the present case, and there will therefore be no singularities in the tensor $F_{\alpha\beta}$.

References

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